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NON-LINEAR LAWS OF FLUID FLOW THROUGH ANISOTROPIC POROUS MEDIA†

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Non-linear laws of fluid flow through anisotropic porous media are written out in invariant tensor form for all crystallographic point symmetry groups. The equations, as is customary in seepage theory [1, 2], are represented by expressions containing the seepage velocity up to and including the third degree. Expressions defining non-linear flow resistances are given and it is shown that, when one transfers from linear to non-linear seepage laws, the symmetry group of the flow properties may change. For example, the isotropic flow properties manifested in Darcy's law may become essentially anisotropic in a non-linear law and display asymmetry, that is, they may be different along one straight line in the positive and negative directions. It is shown that, compared with linear seepage laws for anisotropic media, when flow properties may be defined by just four essentially different types of equation, in non-linear laws the appearance of anisotropy is highly diversified and the number of distinct types of equation increases considerably. © 2002 Elsevier Science Ltd. All rights reserved.

It is well known from experimental data that the range of velocities of a fluid which obeys a linear seepage law – Darcy's law, which defines the relation between the vector fields of the fluid velocity and the pressure gradient – has upper and lower limits [3, 4]. The upper limit of the range in which Darcy's law is applicable is determined by the appearance of inertial forces at high flow velocities, and the lower limit, by physico-chemical effects, due to the fluid interacting with the porous medium and by the non-Newtonian rheological properties of the fluid [5, 6]. Up to the present, however, constructions of non-linear seepage laws have been confined to isotropic porous media. At the same time, it is well known that real soils and collectors of hydrocarbon materials possess anisotropy [4–6]. In this paper, therefore, we will consider versions of the construction of non-linear seepage laws for anisotropic porous media with a given point symmetry group, which will be written out in invariant form for all crystallographic symmetry classes.

1. FUNDAMENTAL ASSUMPTIONS AND FORMULAE

The macroscopic description of flows through porous media is based on the assumption that effective vector fields of the fluid velocity (a vector with components w_i) and pressure gradient (a vector with components $\nabla_i p$) exist, and that these fields satisfy the relation

$$\nabla_i p = f_i(w_i, \rho, \mu, \chi_\alpha, T_\alpha) \tag{1.1}$$

where ρ is the fluid density, μ is its dynamic coefficient of viscosity, χ_α are invariant scalar parameters characterizing the porous medium and possibly the fluid, and T_α are material tensors that determine and define the symmetry of flow resistance.

In the theory of the seepage of a viscous, incompressible, Newtonian fluid in an undeformable porous medium it is assumed that the fluid properties are determined solely by the coefficient of viscosity μ [2–5]. We shall therefore assume from now on that the symmetry of the material tensors T_α in Eq. (1.1) is determined and defined by the symmetry of the pore space. The assumption that the function in Eq. (1.1) is linear leads to Darcy's law. The generalization of the seepage law within the limits of assumption (1.1) implies expansion of the function f_i in a Taylor series in powers of w_i

$$\nabla_i p = -r_{ij} w_j - r_{ijm} w_j w_m - r_{ijmn} w_j w_m w_n \tag{1.2}$$

The material tensors $\|r_{ij}\|$, $\|r_{ijm}\|$ and $\|r_{ijmn}\|$ that define the non-linear flow properties must be invariant under the point group of the pore space; this group is either assumed to be given, when one is solving direct problems, or has to be determined, when one is solving inverse problems. Allowance for quadratic, cubic and higher degrees of approximation, therefore, requires the construction (or definition) of appropriate tensors.

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Using results obtained previously, representation (1.2) may be written down for all point symmetry groups of textures and crystals, apart from cubic terms of the expansion [7], and for the symmetry groups of textures – to within fifth-order terms [8]. However, such a representation is extremely cumbersome and is only an approximation to the functional dependence (1.1). At the same time, it is well known that in the case of isotropic tensor functions, due to the existence of relations among the invariants, the general form of the non-linear relation may be represented by a compact formula in which, instead of constants, there are functions of the invariants [9]. For example, for tensors of rank 2 such a representation is given by the Hamilton–Cayley formula. A solution has been obtained [10] of the problem of constructing generalized Hamilton–Cayley formulae describing the dependence of vector upon vector for the symmetry groups of textures. The general form of the vector potentials and functions consistent with crystal symmetry has been derived [11]; however, the formulae define the potentials and non-linear functions of a vector argument only in a Cartesian system of coordinates, so that the representation is not as general as has been achieved for symmetry groups of textures. On the other hand, the formulae frequently involve arguments of fourth and higher degree, that is, they presuppose a degree of accuracy which, in applied problems of subsurface hydromechanics, yields no quantitative or qualitative modification of the result, merely complicating its derivation. As already remarked, in order to represent non-linear laws in seepage theory for isotropic porous media one uses equations that generally contain the fluid velocity up to the second degree and rarely the third. Therefore, having retained the formulae in representations of the vector functions of [11] that contain the vector argument only up to the third degree, and using systems of governing parameters characterizing the geometrical properties of anisotropic media [7], one obtains the non-linear governing equations of seepage theory for crystal symmetry groups in invariant tensor form.

To illustrate the transformations carried out when one transfers from the representation of a non-linear tensor relation in a Cartesian system of coordinates to an invariant tensor form, we will consider two simple examples for the symmetry groups $6/4$ and $3/4$ of a cube (cubic crystal system) (the notation used here for symmetry groups is that of Shubnikov).

The symmetry group $3/4$ is defined [11] by principal invariants of the form w_iw_i, $w_1w_2w_3$, $w_1^4 + w_2^4 + w_3^4$, and the non-linear vector function is given by the formula

$$\begin{aligned}\operatorname{grad} p = &-f_1(w_1\mathbf{e}_1 + w_2\mathbf{e}_2 + w_3\mathbf{e}_3) - f_2(w_2w_3\mathbf{e}_1 + w_3w_1\mathbf{e}_2 + w_1w_2\mathbf{e}_3) - \\ &-f_3(w_1^3\mathbf{e}_1 + w_2^3\mathbf{e}_2 + w_3^3\mathbf{e}_3)\end{aligned} \tag{1.3}$$

where f_i are arbitrary functions of the principal invariants. As simple tensors determining and defining the geometrical properties of the symmetry group $3/4$ the following tensors were used [7]

$$\begin{aligned}&\mathbf{g} = \mathbf{e}_1^2 + \mathbf{e}_2^2 + \mathbf{e}_3^2, \quad T_d = \mathbf{e}_1\mathbf{e}_2\mathbf{e}_3 + \mathbf{e}_2\mathbf{e}_1\mathbf{e}_3 + \mathbf{e}_2\mathbf{e}_3\mathbf{e}_1 + \mathbf{e}_3\mathbf{e}_2\mathbf{e}_1 + \mathbf{e}_3\mathbf{e}_1\mathbf{e}_2 + \mathbf{e}_1\mathbf{e}_3\mathbf{e}_2 \\ &\mathbf{O}_h = \mathbf{e}_1^4 + \mathbf{e}_2^4 + \mathbf{e}_3^4\end{aligned}$$

in which, as in (1.3), $\mathbf{e}_i$ are the unit vectors of the crystal physics Cartesian basis; powers of basis vectors and the expressions themselves are understood as dyadic and polyadic products; here and below, the notation proposed in [7] is used for all simple (basic) tensors.

In terms of the basis tensors $\mathbf{g}$, $\mathbf{T}_d$, $\mathbf{O}_h$, formula (1.3) may be rewritten as

$$\nabla_i p = -f_1 w_i - f_2 T_{(d)ijk} w_j w_k - f_3 O_{(h)ijkl} w_j w_k w_l \tag{1.4}$$

Formula (1.4), unlike Eq. (1.3), is a non-linear seepage law in invariant tensor form, and it may be written in any system of coordinates. Note that the same seepage law is obtained for symmetry group $2/3$ of the cubic system.

Symmetry group $6/4$ is defined by principal vector invariants

$$w_iw_i, \quad w_1w_2 + w_3w_1 + w_2w_3, \quad w_1^2w_2^2w_3^2$$

and the non-linear vector function is given by the equality

$$\begin{aligned}\operatorname{grad} p = &-f_1(w_1\mathbf{e}_1 + w_2\mathbf{e}_2 + w_3\mathbf{e}_3) - f_2(w_1^3\mathbf{e}_1 + w_2^3\mathbf{e}_2 + w_3^3\mathbf{e}_3) - \\ &-f_3w_1w_2w_3(w_2w_3\mathbf{e}_1 + w_1w_3\mathbf{e}_2 + w_1w_2\mathbf{e}_3)\end{aligned} \tag{1.5}$$

where f_i are arbitrary functions of the principal invariants. As simple tensors determining and defining the geometrical properties of the symmetry group $^6/_4$, the tensors $\mathbf{g}$, $\mathbf{T}_d$, $\mathbf{O}_h$ were used [7]. Using the simple (basis) tensors, formula (1.5) may be rewritten as

$$\nabla_i p = -f_1 w_i - f_2 O_{(h)ijkl} w_j w_k w_l - f_3 T_{(d)ijk} T_{(d)lmn} w_j w_k w_l w_m w_n \tag{1.6}$$

or, ignoring the last term

$$\nabla_i p = -f_1 w_i - f_2 O_{(h)ijkl} w_j w_k w_l \tag{1.7}$$

The governing equations for symmetry groups $^6/_2$ and $^3/_4$ of the cubic system also have the form (1.7) up to the third power of the fluid velocity.

Similar arguments yield invariant tensor representations of non-linear seepage laws for the other crystal symmetry groups also. Therefore, omitting the details, we will present the explicit form of the governing relations for the remaining symmetry groups. The corresponding symmetry group is indicated below in braces; $\nabla_i p(\mathrm{m}\cdot 4:\mathrm{m})$ denotes the right-hand side of the relation for the symmetry group $m\cdot 4:m$, and so on.

Tetragonal system

$$\nabla_i p = -f_1 w_i - f_2 B_{ij} w_j - f_3 O_{(h)ijkl} w_j w_k w_l \qquad \{\mathrm{m}\cdot 4:\mathrm{m}, 4:2\}$$

$$\nabla_i p = \nabla_i p(\mathrm{m}\cdot 4:\mathrm{m}) - f_4 T_{(d)ijk} w_j w_k \qquad \{\bar{4}\cdot\mathrm{m}\}$$

$$\nabla_i p = \nabla_i p(\mathrm{m}\cdot 4:\mathrm{m}) - f_4 O_{(h)ijkl} \Omega_{lm} w_j w_k w_m \qquad \{4:\mathrm{m}\}$$

$$\nabla_i p = \nabla_i p(4:\mathrm{m}) - f_5 B_{ij} T_{(d)jkl} w_k w_l \qquad \{\bar{4}\}$$

$$\nabla_i p = -f_1 b_i - f_2 w_i - f_3 O_{(h)ijkl} w_j w_k w_l \qquad \{4\cdot\mathrm{m}\}$$

$$\nabla_i p = \nabla_i p(4\cdot\mathrm{m}) - f_4 O_{(h)ijkl} \Omega_{lm} w_j w_k w_m \qquad \{4\}$$

Trigonal and hexagonal system

$$\nabla_i p = -f_1 w_i - f_2 B_{ij} w_j \qquad \{\mathrm{m}\cdot 6:\mathrm{m}, 6:\mathrm{m}, 6:2, \bar{6}\cdot\mathrm{m}\}$$

$$\nabla_i p = -f_1 b_i - f_2 g_{ij} w_j \qquad \{6, 6\cdot\mathrm{m}\}$$

$$\nabla_i p = -f_1 w_j - f_2 B_{ij} w_j - f_3 D_{(3h)ijk} w_j w_k \qquad \{\mathrm{m}\cdot 3:\mathrm{m}, 3:2\}$$

$$\nabla_i p = \nabla_i p(\mathrm{m}\cdot 3:\mathrm{m}) - f_4 D_{(3h)ijk} \Omega_{kl} w_j w_l \qquad \{3:\mathrm{m}\}$$

$$\nabla_i p = \nabla_i p(\mathrm{m}\cdot 6:\mathrm{m}) - f_3 D_{(3d)ijkl} w_j w_k w_l \qquad \{\bar{6}\}$$

$$\nabla_i p = \nabla_i p(6\cdot\mathrm{m}) - f_3 D_{(3h)ijk} \Omega_{kl} w_j w_l \qquad \{3\cdot\mathrm{m}\}$$

$$\nabla_i p = \nabla_i p(3\cdot\mathrm{m}) - f_4 D_{(3h)ijk} w_j w_k - f_5 b_i D_{(3h)jkl} w_j w_k w_l \qquad \{3\}$$

Note that the formulae for the symmetry groups $\mathrm{m}\cdot 6:\mathrm{m}$, $6:\mathrm{m}$, $6:2$, $\bar{6}\cdot\mathrm{m}$ of the tetragonal and hexagonal system are described by non-linear seepage laws identical with those for textures that have the symmetry of a cylinder, while the symmetry groups 6 and $6\cdot\mathrm{m}$ are described by non-linear seepage laws identical with those for textures that have the symmetry of a cone.

Rhombic system

$$\nabla_i p = -f_1 w_i - f_2 D_{(2h)ij} w_j - f_3 M_{ij} w_j \qquad \{\mathrm{m}\cdot 2:\mathrm{m}\}$$

$$\nabla_i p = \nabla_i p(\mathrm{m}\cdot 2:\mathrm{m}) - f_4 D_{(2h)ij} E_{ljm} M_{mk} w_j w_k \qquad \{2:2\}$$

$$\nabla_i p = -f_1 b_i - f_2 D_{(2h)ij} w_j - f_3 M_{ij} w_j \qquad \{2\cdot\mathrm{m}\}$$

Monoclinic system

$$\nabla_i p = \nabla_i p(2\cdot\mathrm{m}) - f_4\Omega_{ik}D_{(2h)kj}w_j - f_5\Omega_{ik}D_{(2h)kj}b_l w_j w_l \qquad \{2\}$$

$$\nabla_i p = -f_1 a_i - f_2 c_i - f_3 D_{(2h)ij}w_i \qquad \{\mathrm{m}\}$$

$$\nabla_i p = \nabla_i p(\mathrm{m}\cdot 2:\mathrm{m}) - f_4\Omega_{ik}D_{(2h)kl}w_l - f_5 D_{(2h)ij}\Omega_{km}D_{(2h)ml}w_j w_k w_l \qquad \{2:\mathrm{m}\}$$

Triclinic system

$$\nabla_i p = \nabla_i p(2:\mathrm{m}) - f_6\omega_{ik}^{(3)}D_{(2h)kj}w_j - f_7\omega_{ik}^{(2)}D_{(2h)kj}w_j -$$

$$-f_8 D_{(2h)ij}\omega_{km}^{(2)}D_{(2h)ml}w_j w_k w_l - f_9 D_{(2h)ij}\omega_{km}^{(3)}D_{(2h)ml}w_j w_k w_l \qquad \{\bar{2}\}$$

$$\nabla_i p = -f_1 a_i - f_2 c_i - f_3 b_i \qquad \{1\}$$

To simplify the notation, the seepage laws have been written in a Cartesian system of coordinates, and the symmetrization operation has not been indicated; it is therefore assumed that all the tensors are symmetric with respect to all subscripts. In the general case, this means that the coefficient f_i is equal to the sum of tensors obtained by permuting the subscripts in the symmetrization operation.

2. DEFINITION OF FLOW PROPERTIES

Anisotropic flow properties along the direction defined by a unit vector with components n_i are defined by relations of the following type [1]

$$k(n) = -w_i n_i / \nabla p \quad \text{or} \quad r(n) = -\nabla_i p n_i / w \tag{2.1}$$

where w and ∇p are the moduli of the seepage velocity vector and the pressure gradient, respectively. The first equation of (2.1) determines the value of the flow resistance coefficient when the direction of the unit vector with components n_i coincides with that of the pressure gradient; the second determines the value of the flow resistance when the direction of the unit vector coincides with that of the seepage velocity. Substitution of the non-linear seepage laws written out above into the second equation of (2.1) gives the explicit form of the flow resistance for all symmetry groups of textures and crystals.

As an example, we shall consider only (1.4) and (1.7) for the symmetry groups of the cubic system.

Substitution of the non-linear seepage law (1.4) into (2.1) yields the following expression for the flow resistance

$$r(n) = f_1 + 3f_2 n_1 n_2 n_3 w + f_3(n_1^4 + n_2^4 + n_3^4)w^2 \tag{2.2}$$

where n_i are the components of the unit vector, defined in the crystal physics system of coordinates.

For the non-linear governing equations (1.7), the expression for the flow resistance is

$$r(n) = f_1 + f_2(n_1^4 + n_2^4 + n_3^4)w^2 \tag{2.3}$$

Formulae (2.2) and (2.3) differ only in the presence of a term containing the first power of the seepage velocity. However, it is precisely the presence of that term in (2.2) that causes the flow resistance for symmetry groups $\bar{3}/4$ and $\bar{2}/3$ to possess asymmetry of the flow properties, that is to say, if the flow direction is reversed, the flow resistance will not have the same values in the "forward" and "reverse" directions. The asymmetry effect is observed for all directions in which $n_1 n_2 n_3 \neq 0$.

The effect of asymmetry of the flow properties is observed for all symmetry groups whose governing equations involve tensors of odd rank (these are the symmetry groups $4\cdot\mathrm{m}$, $\bar{4}\cdot\mathrm{m}$, $\bar{4}$, 4, $\mathrm{m}\cdot 3:\mathrm{m}$, $3:2$, $3:\mathrm{m}$, $3\cdot\mathrm{m}$, 3, 6, $6\cdot\mathrm{m}$, $2:2$, $2\cdot\mathrm{m}$, 2, m, 1).

An important observation here is that, in the crystal symmetry groups of the cubic system considered here as an example, the flow properties in the linear governing equations are isotropic for all symmetry groups. Thus, a change from linear governing equations to non-linear ones changes the symmetry group of the physical properties.

The change in the symmetry group of flow resistance (permeability) may cause a substantial change in the techniques used to perform experimental research and interpret experimental data. Indeed, for porous media exhibiting isotropic flow properties in Darcy's law, any direction is the principal one for the tensor of the flow resistance coefficients (permeability). Therefore, in a laboratory determination of flow properties, the specimen may be oriented in an arbitrary way with respect to the crystal physics system of coordinates; nevertheless, the characteristics obtained as a result of measurements may be processed by standard techniques and they yield the value of the coefficient of absolute permeability (flow resistance).

However, if anisotropy appears on changing to describing the non-linear properties of the medium, the selected direction may prove to be non-"principal" (in the sense of the vectors $\nabla_i p$ and w_i being parallel). The measured characteristics, if processed by standard techniques, will then yield the "effective permeability" and will depend on the ratio of the diameter of the specimen to its length [13]. In fact, if the seepage flow takes place along a non-principal direction, the measured flow rate will depend on the relation between the length and diameter of the specimen. Under such conditions the "effective permeability" will vary from the value of the directional permeability (for a thin plate) to the reciprocal of the directional flow resistance (for a long rod) [13]. Such an experiment has indeed been described [14], but because the medium was assumed to be isotropic even in the context of describing non-linear properties, the effect was not confirmed. In particular, for the symmetry groups of the cubic system the "principal" directions in the non-linear governing equations are the directions of the crystal physics coordinate axes – the directions of the edges of a cube representing the unit cell of the crystal [15] and directions coinciding with those of the diagonals of the cube.

Similar effects, due to the change in the symmetry of flow resistance (permeability) on changing from linear governing equations to non-linear ones, are also observed for symmetry groups of crystals in the tetragonal, hexagonal and trigonal systems. In the case of linear governing equations, the flow properties of all these crystal symmetry groups are identical with those of anisotropic textures. The flow properties of anisotropic textures are characterized by the presence of a plane of isotropy of the flow properties (and they are consequently often called transversally isotropic textures). When one changes from linear to non-linear governing equations, all the crystal symmetry groups of the tetragonal system have governing equations different from those for anisotropic textures. Moreover, in none of the symmetry groups is the isotropy plane of the flow properties preserved. Once again, as in the case of the symmetry groups of the cubic system, the "principal" directions are still the directions of the axes of the crystal physics system of coordinates.

Half of the symmetry groups of the hexagonal and trigonal systems possess non-linear governing equations analogous to the equations of anisotropic textures ($m \cdot 6 : m$, $6 : m$, $6 : 2$, $\bar{6} : m$ are textures with the symmetry of a cylinder and 6, $m \cdot 6$ are textures of the symmetry of a cone). The other half of the symmetry groups possess non-linear governing equations different from those for textures (symmetry groups $m \cdot 3 : m$, $3 : 2$, $3 : m$, $\bar{6}$, $3 \cdot m$, 3). Consequently, in a laboratory determination of flow properties, versions may appear in these cases as well in which the characteristics determined are not true but "effective".

For the symmetry groups of the rhombic, monoclinic and triclinic systems, changing from linear to non-linear governing equations produces an effect of asymmetry of the flow properties (symmetry groups $2 : 2$, $2 \cdot m$, 2, m and 1) and different governing equations (Darcy's laws for all groups of each of the above-mentioned systems were identical).

Depending on the soil structure (oriented, ordered, chaotic [6]) and the type of collector (porous, fractured, etc. [4, 5]), real porous media may possess a variety of local symmetries of the pore space and correspond to either type of formula as presented above.

3. REPRESENTATION OF NON-LINEAR SEEPAGE LAWS

The explicit form of the functions of the invariants f_i in the non-linear generalized seepage laws, and the values of the tensor components in expansion (1.2), may be determined by processing experimental data. Assuming that the seepage flow properties of the medium are isotropic, the present experimental data are satisfactorily approximated by formulae of the following form [1]

$$\nabla p = aw + bw^2 \quad \text{or} \quad \nabla p = aw + bw^2 + cw^3 \tag{3.1}$$

the first of which is known as Forchheimer's formula. One may therefore expect that the non-linear seepage laws for anisotropic media may also be represented by similar formulae, especially as in the course of experimental measurements it is almost always assumed a priori, without proof, that the properties are isotropic. In generalized representations of non-linear seepage laws one then stipulates a class of functions determining and defining seepage flow properties (polynomials formed by convoluting the basis tensors with the flow velocity vector) and their order. The representations of non-linear seepage laws written out above involve certain functions of the invariants that may be coefficients of basis tensors of ranks one, two, three and four (for example, $fb_i, fB_{ij}w_j, fT_{(d)ijk}w_jw_k, fO_{(h)ijkl}w_jw_kw_l$). According to our assumptions, we deduce that all functions that may be coefficients of the basis tensors of rank four (or of combinations of such tensors) are constants. The functions that may be coefficients of material tensors (a_i, b_i, c_i) of rank one for symmetry groups 4 · m, 4, 6 · m, 6, m, 2 · m, 2 and 1 may be expressed as

$$f = ab_iw_i + b(b_iw_i)^2 + c(b_iw_i)^3 \tag{3.2}$$

where a, b and c are constants. Formula (3.2) represents a function that may be the coefficient of a material vector with components b_i. It is obvious that functions that may be coefficients of the vectors with components a_i and c_i are analogous in form.

The representation of the "isotropic" term fw_i in non-linear seepage laws may be taken as

$$f = a + bw \tag{3.3}$$

The following remark should be noted. Representing a non-linear seepage law in the form (1.2) does not lead to Forchheimer's formula (3.1) for an isotropic porous medium, since the quadratic term in the law is due to a tensor of rank three, which vanishes identically. Nevertheless, this discrepancy between experimental results and formula (1.2) is easily eliminated if one resorts to generalized formulae for non-linear seepage laws. In fact, the generalized non-linear seepage law for isotropic porous media has the form [11]

$$\nabla_i p = -fw_i, \quad f = f(w^2) \tag{3.4}$$

Therefore, processing of experimental results on the assumption that $f = a + bw$ leads to Forchheimer's seepage law.

Functions appearing as coefficients of the tensors $\|B_{ij}\|$, $\|D_{(2h)ij}\|$, $\|\Omega_{ik}D_{(2h)kj}\|$ may be expressed either as constants or, by analogy with the isotropic term with inertial "corrections," as constants multiplied by invariants, formed by contracting tensors with the flow velocity vector. Functions defining the asymmetry of flow properties may be given as constants.

The representations obtained here for non-linear seepage laws in anisotropic media possessing crystal symmetry, as well as the accompanying analysis, indicate that the study of non-linear flows through porous media should yield additional information on the structure of the pore space.

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